How much Energy is Converted from the Wind by a Wind Turbine?

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1) The power in the wind is proportional to the cube of the wind speed
i.e. double the *wind speed* and the power will increase by a factor of eight (2 x 2 x 2).
2) The power output of a wind turbine is proportional to the area swept by the rotor – i.e. double the *swept area* and the power output is also doubled. (The forces on a wind turbine at higher wind speeds can get very high so its output will be limited at a rated power. Therefore, at a site with double the average mean wind speed, won't generate eight times the energy but it would generate over four times.)

1. Simple Calculation of Wind Power

Wind is made up of moving air molecules which have mass - though not much. Any moving object with mass carries **kinetic energy** in an amount which is given by the equation:

$$\textbf{Kinetic Energy} = 0.5 \times \textbf{Mass} \times \textbf{Velocity}^2$$

The units of measurement are:

mass - **kg**,
velocity - **m/s**
energy - **joules**.

To calculate the **mass** of air flowing into the turbine **per second**, use the following equation:

$$\textbf{Mass/sec (kg/s)} = \textbf{Velocity (m/s)} \times \textbf{Area } (m^2) \times \textbf{Density } (kg/m^3)$$

(On average in the UK, air has a known density of about 1.225 kg/m^3 at sea level)

And therefore, the **power** (i.e. energy per second) in the wind flowing into a wind turbine with a certain swept area is given by inserting the *mass per second* calculation into the standard *kinetic energy* equation (above) resulting in the following **vital** equation:

$$\textbf{Power} = 0.5 \times \textbf{Swept Area} \times \textbf{Air Density} \times \textbf{Velocity}^3$$

The units of measurement are:

Power - **Watts (i.e. joules/second)**
Swept area - **square metres**
Air density - **kilograms per cubic metre**
Velocity - **metres per second**

2. Actual Energy Extracted from the Wind by a Wind Turbine

Wind Power Calculation in the Real World
Using the example of one of the largest wind turbines in the world with a rotor blade diameter of 126 metres, the rotors sweep an area of PI x (diameter/2)^2 = 12470 m^2. This is an off-shore wind turbine in the UK, so it is situated at sea-level and therefore the air density is 1.225 kg/m^3. The turbine is rated at 5MW in 30mph (14m/s) winds, and so, by putting in the known values to the *power equation* (above), gives

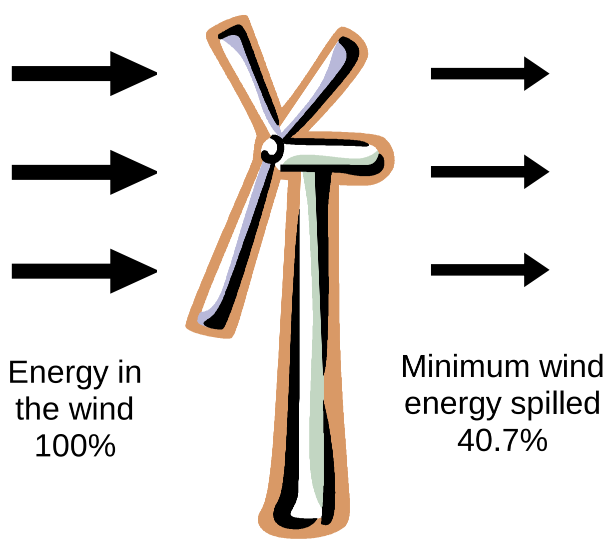
Wind Power = 0.5 x 12,470 x 1.225 x (14 x 14 x 14)

...which calculates the power in the wind of around 21,000,000 Watts (21MW), - much more than the rated power of the turbine generator (5MW) because of the Betz Limit: that at 14 m/s the fraction of power the turbine is extracting has been reduced so as to reduce the forces on the turbine itself, as well as inefficiencies in the system, see below.

No turbine can capture more than 16/27 (59.3%) of the kinetic energy of the wind according to German physicist, Albert Betz. This limit is known as the **Betz Limit** and has nothing to do with inefficiencies in the generator, but how turbines actually work.

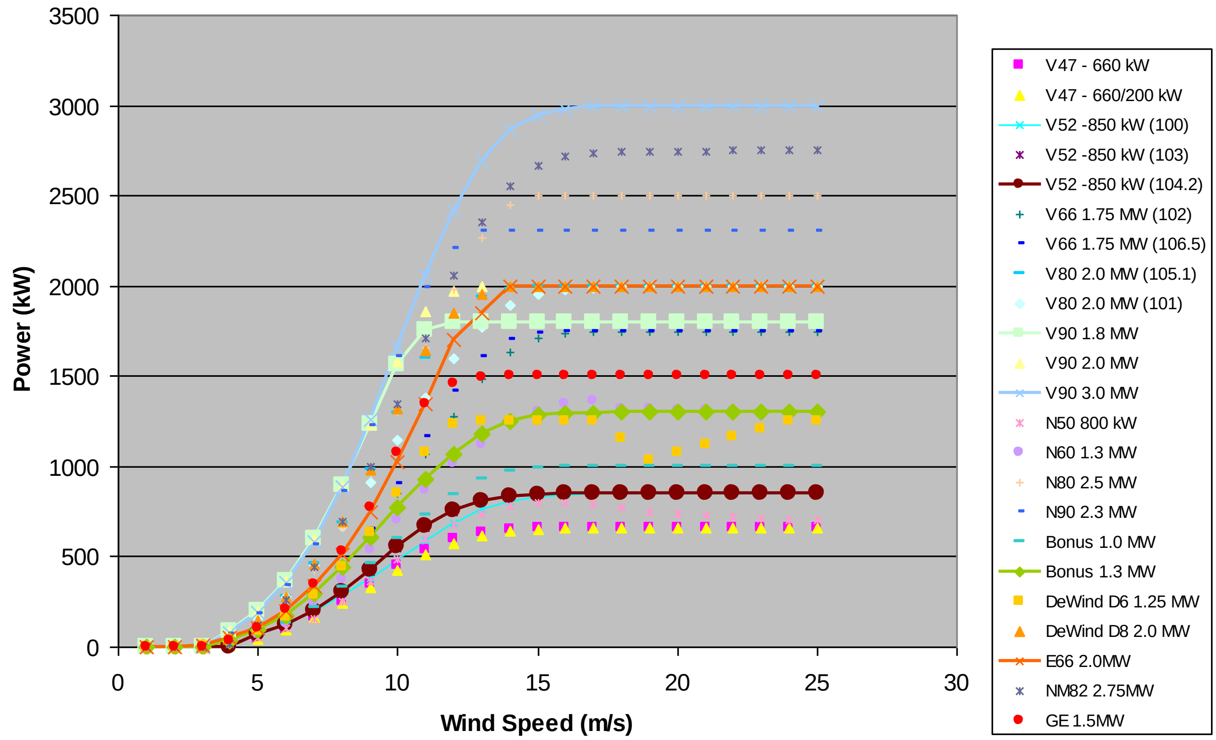
Wind turbines extract energy and so slow down the wind passing through them. For a wind turbine to extract 100% of the energy it would need to stop 100% of the wind. If it did this it would then stop working. Instead it extracts enough energy to still allow the wind to continue to pass through it.

Betz Limit; A theoretical maximum of 59.3% of the wind energy can be extracted

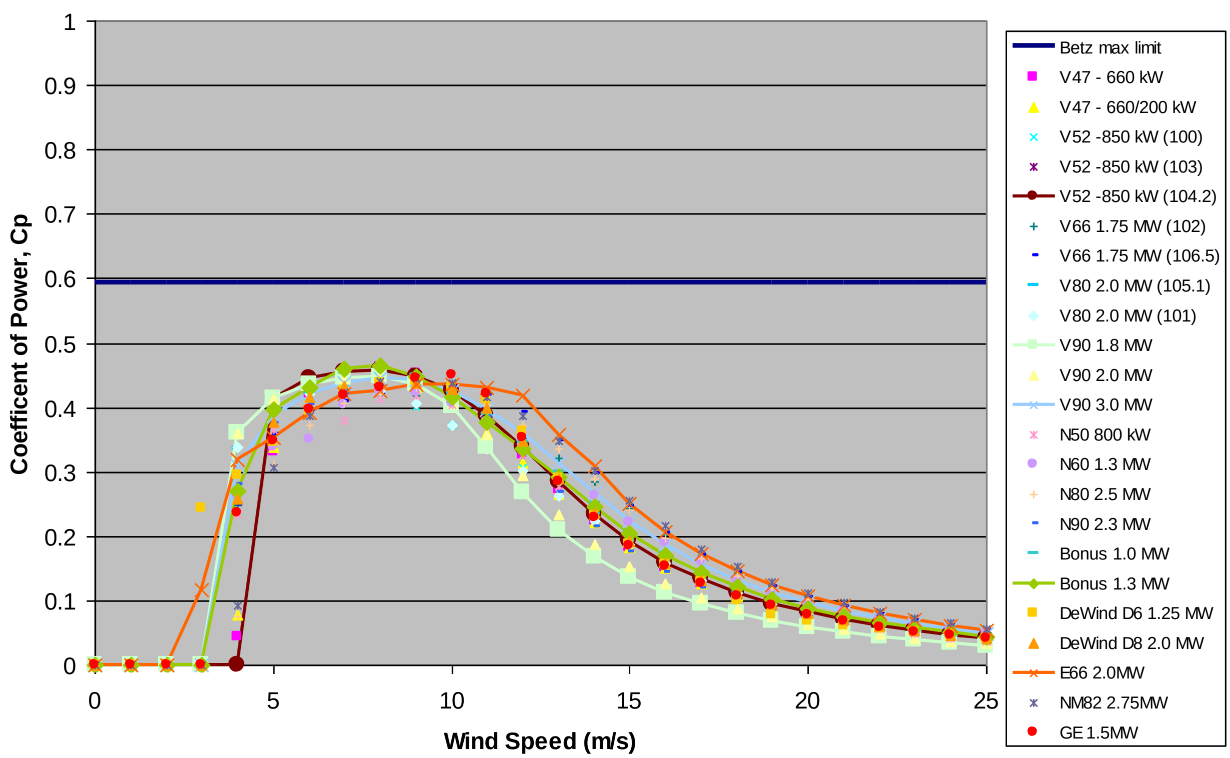


The Betz limit is an over estimate of the maximum energy extracted. British aerodynamicist Hermann Glauert's Blade Element Momentum Theory (1935) gives a lower theoretical maximum and more detailed models lower still. Practically, and under certain conditions, the most efficient modern turbine can extract close to 50% of the power in the wind after losses in a complete wind turbine system, including the blades, generator, bearings power conversion and transmission.

The figure below shows some typical power curves presenting the different rated powers where the power output is limited to limit the forces on the turbine.



The figure below shows how much of the power is extracted at different wind speeds.



Once the engineering requirements of limiting the total output power for reasons of strength and durability, are included, the real limit is well below the theoretical limit and depends on the spread of wind speed seen at a specific site. Westmill's turbines extract about 32% of the energy in the wind passing through them in a year.

More usually the output of a turbine at a specific site is expressed as a **capacity factor** which is the ratio of the actual energy generated (in a year) to the theoretical but totally unreal amount that would have been generated if the turbine had been generating at rated power throughout (a year). Westmill's annual capacity factor is about 21% but this does vary year on year depending on the wind that actually blows during each year.

The Burradale windfarm outside Lerwick, Shetland has the world record of a capacity factor of 57.9% in 2005. However because it is so windy there the limiting of the peak power output from the turbines will have meant that they only extracted about 15% of the energy in the wind which is less than at Westmill.

Can you use the power equation, Betz limit and the capacity factor to calculate a Westmill Turbine's Power?
You will need to find out the blade length and the current wind speed. (Assume sea level).

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Web Links
http://en.wikipedia.org/wiki/Betz%27_law